CHAPTER 12 Differentiation

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. If $y = f\left(\frac{2x-1}{x^2+1}\right)$ and $f'(x) = \sin x^2$, then $\frac{dy}{dx} =$ .... *(1982 - 2 Marks)*

2. If $f_r(x), g_r(x), h_r(x)$, $r = 1, 2, 3$ are polynomials in x such that $f_r(a) = g_r(a) = h_r(a), r = 1, 2, 3$

and $F(x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix}$ then $F'(x)$ at $x = a$ is .... *(1985 - 2 Marks)*

3. If $f(x) = \log_x(\ln x)$, then $f'(x)$ at $x = e$ is .... *(1985 - 2 Marks)*

4. The derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ with respect to $\sqrt{1-x^2}$ at $x = \frac{1}{2}$ is .... *(1986 - 2 Marks)*

5. If $f(x) = |x-2|$ and $g(x) = f[f(x)]$, then $g'(x) =$ .... for $x > 20$ *(1990 - 2 Marks)*

6. If $xe^{xy} = y + \sin^2 x$, then at $x = 0$, $\frac{dy}{dx} =$ .... *(1996 - 1 Mark)*

B True/ False

1. The derivative of an even function is always an odd function. *(1983 - 1 Mark)*

C MCQs with One Correct Answer

1. If $y^2 = P(x)$, a polynomial of degree 3, then $2\frac{d}{dx}\left(y^3 \frac{d^2y}{dx^2}\right)$ equals *(1988 - 2 Marks)*
   (a) $P'''(x) + P'(x)$ (b) $P''(x)P'''(x)$
   (c) $P(x)P'''(x)$ (d) a constant

2. Let $f(x)$ be a quadratic expression which is positive for all the real values of x. If $g(x) = f(x) + f'(x) + f''(x)$, then for any real x, *(1990 - 2 Marks)*
   (a) $g(x) < 0$ (b) $g(x) > 0$
   (c) $g(x) = 0$ (d) $g(x) \geq 0$

3. If $y = (\sin x)^{\tan x}$, then $\frac{dy}{dx}$ is equal to *(1994)*
   (a) $(\sin x)^{\tan x}(1 + \sec^2 x \log \sin x)$
   (b) $\tan x (\sin x)^{\tan x - 1}.\cos x$
   (c) $(\sin x)^{\tan x} \sec^2 x \log \sin x$
   (d) $\tan x (\sin x)^{\tan x - 1}$

4. If $x^2 + y^2 = 1$ then *(2000)*
   (a) $yy'' - 2(y')^2 + 1 = 0$ (b) $yy'' + (y')^2 + 1 = 0$
   (c) $yy'' + (y')^2 - 1 = 0$ (d) $yy'' + 2(y')^2 + 1 = 0$

5. Let $f : (0, \infty) \to R$ and $F(x) = \int_0^x f(t)dt$. If $F(x^2) = x^2(1+x)$, then $f(4)$ equals *(2001S)*
   (a) 5/4 (b) 7 (c) 4 (d) 2

6. If y is a function of x and $\log(x+y) - 2xy = 0$, then the value of $y'(0)$ is equal to *(2004S)*
   (a) 1 (b) -1 (c) 2 (d) 0

7. If $f(x)$ is a twice differentiable function and given that $f(1) = 1; f(2) = 4, f(3) = 9$, then *(2005S)*
   (a) $f''(x) = 2$ for $\forall\ x \in (1, 3)$
   (b) $f''(x) = f'(x) = 5$ for some $x \in (2, 3)$
   (c) $f''(x) = 3$ for $\forall\ x \in (2, 3)$
   (d) $f''(x) = 2$ for some $x \in (1, 3)$

8. $\frac{d^2x}{dy^2}$ equals *(2007 -3 marks)*
   (a) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (b) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$
   (c) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (d) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

9. Let $g(x) = \log f(x)$ where $f(x)$ is twice differentible positive function on $(0, \infty)$ such that $f(x+1) = x f(x)$. Then, for $N = 1, 2, 3, \ldots\ldots\ldots$ *(2008)*

$$g''\left(N+\frac{1}{2}\right)-g''\left(\frac{1}{2}\right)=$$

(a) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots..+\frac{1}{(2N-1)^2}\right\}$

(b) $4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots..+\frac{1}{(2N-1)^2}\right\}$

(c) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots..+\frac{1}{(2N+1)^2}\right\}$

(d) $4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots..+\frac{1}{(2N+1)^2}\right\}$

10. Let $f : [0, 2] \to R$ be a function which is continuous on $[0, 2]$ and is differentiable on $(0, 2)$ with $f(0) = 1$. Let $F(x) = \int_0^{x^2} f\left(\sqrt{t}\right)dt$ for $x \in [0,2]$. If $F'(x) = f'(x)$ for all $x \in (0,2)$, then $F(2)$ equals *(JEE Adv. 2014)*

(a) $e^2 - 1$ (b) $e^4 - 1$
(c) $e - 1$ (d) e^4

D MCQs with One or More than One Correct

1. Let $f : \mathbb{R} \to \mathbb{R}$, $g : \mathbb{R} \to \mathbb{R}$ and $h : \mathbb{R} \to \mathbb{R}$ be differentiable functions such that $f(x) = x^3 + 3x + 2$, $g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in \mathbb{R}$. Then *(JEE Adv. 2016)*

(a) $g'(2) = \frac{1}{15}$ (b) $h'(1) = 666$
(c) $h(0) = 16$ (d) $h(g(3)) = 36$

E Subjective Problems

1. Find the derivative of $\sin(x^2 + 1)$ with respect to x from first principle. *(1978)*

2. Find the derivative of

$$f(x) = \begin{cases} \dfrac{x-1}{2x^2-7x+5} & \text{when } x \neq 1 \\ -\dfrac{1}{3} & \text{when } x = 1 \end{cases}$$

at $x = 1$ *(1979)*

3. Given $y = \frac{5x}{3\sqrt{(1-x)^2}} + \cos^2(2x+1)$; Find $\frac{dy}{dx}$. *(1980)*

4. Let $y = e^{x \sin x^3} + (\tan x)^x$. Find $\frac{dy}{dx}$ *(1981 - 2 Marks)*

5. Let f be a twice differentiable function such that

$f''(x) = -f(x)$, and $f'(x) = g(x)$, $h(x) = [f(x)]^2 + [g(x)]^2$

Find $h(10)$ if $h(5) = 11$ *(1982 - 3 Marks)*

6. If α be a repeated root of a quadratic equation $f(x) = 0$ and $A(x)$, $B(x)$ and $C(x)$ be polynomials of degree 3, 4 and 5 respectively, then show that $\begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$ is divisible by $f(x)$, where prime denotes the derivatives. *(1984 - 4 Marks)*

7. If $x = \sec\theta - \cos\theta$ and $y = \sec^n\theta - \cos^n\theta$, then show that $(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$ *(1989 - 2 Marks)*

8. Find $\frac{dy}{dx}$ at $x = -1$, when

$$(\sin y)^{\sin\left(\frac{\pi}{2}x\right)} + \frac{\sqrt{3}}{2}\sec^{-1}(2x) + 2^x \tan(\ln(x+2)) = 0$$

(1991 - 4 Marks)

9. If $y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{x-c} + 1$, prove that $\frac{y'}{y} = \frac{1}{x}\left(\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right)$. *(1998 - 8 Marks)*

H Assertion & Reason Type Questions

1. Let $f(x) = 2 + \cos x$ for all real x.

STATEMENT - 1 : For each real t, there exists a point c in $[t, t+\pi]$ such that $f'(c) = 0$ because

STATEMENT - 2 : $f(t) = f(t + 2\pi)$ for each real t. *(2007 -3 marks)*

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True.

2. Let f and g be real valued functions defined on interval $(-1, 1)$ such that $g''(x)$ is continuous, $g(0) \neq 0$. $g'(0) = 0$, $g''(0) \neq 0$, and $f(x) = g(x) \sin x$

STATEMENT - 1 : $\lim_{x \to 0} [g(x) \cot x - g(0) \operatorname{cosec} x] = f''(0)$

and

STATEMENT - 2 : $f'(0) = g(0)$ *(2008)*

(a) Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for Statement - 1
(b) Statement - 1 is True, Statement - 2 is True; Statement - 2 **is NOT** a correct explaination for Statement - 1
(c) Statement - 1 is True, Statement - 2 is False
(d) Statement - 1 is False, Statement - 2 is True

I Integer Value Correct Type

1. If the function $f(x) = x^3 + e^{\frac{x}{2}}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is *(2009)*

2. Let $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$.

Then the value of $\frac{d}{d(\tan\theta)}(f(\theta))$ is *(2011)*

Section-B JEE Main / AIEEE

1. If $y = (x + \sqrt{1 + x^2})^n$, then $(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is **[2002]**

(a) n^2y (b) $-n^2y$ (c) $-y$ (d) $2x^2y$

2. If $f(y) = e^y$, $g(y) = y$; $y > 0$ and

$F(t) = \int_0^t f(t-y)g(y)\,dy$, then **[2003]**

(a) $F(t) = te^{-t}$ (b) $F(t) = 1 - te^{-t}(1+t)$

(c) $F(t) = e^t - (1+t)$ (d) $F(t) = te^t$.

3. If $f(x) = x^n$, then the value of **[2003[**

$f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} + \ldots\ldots\ldots \frac{(-1)^n f^n(1)}{n!}$ is

(a) 1 (b) 2^n (c) $2^n - 1$ (d) 0

4. Let $f(x)$ be a polynomial function of second degree. If $f(1) = f(-1)$ and a, b, c are in A. P, then $f'(a), f'(b), f'(c)$ are in **[2003]**

(a) Arithmetic -Geometric Progression
(b) A.P
(c) G.P
(d) H.P.

5. If $x = e^{y + e^{y + e^{y + \ldots\infty}}}$, $x > 0$, then $\frac{dy}{dx}$ is **[2004]**

(a) $\frac{1+x}{x}$ (b) $\frac{1}{x}$ (c) $\frac{1-x}{x}$ (d) $\frac{x}{1+x}$

6. The value of a for which the sum of the squares of the roots of the equation $x^2 - (a-2)x - a - 1 = 0$ assume the least value is **[2005]**

(a) 1 (b) 0 (c) 3 (d) 2

7. If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals **[2005]**

(a) -2 (b) 3 (c) 2 (d) 1

8. Let $f: R \to R$ be a differentiable function having $f(2) = 6$, $f'(2) = \left(\frac{1}{48}\right)$. Then $\lim_{x \to 2} \int_6^{f(x)} \frac{4t^3}{x-2} dt$ equals **[2005]**

(a) 24 (b) 36 (c) 12 (d) 18

9. The set of points where $f(x) = \frac{x}{1+|x|}$ is differentiable is

(a) $(-\infty, 0) \cup (0, \infty)$ (b) $(-\infty, -1) \cup (-1, \infty)$
(c) $(-\infty, \infty)$ (d) $(0, \infty)$ **[2006]**

10. If $x^m . y^n = (x+y)^{m+n}$, then $\frac{dy}{dx}$ is **[2006]**

(a) $\frac{y}{x}$ (b) $\frac{x+y}{xy}$ (c) xy (d) $\frac{x}{y}$

11. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then $y'(1)$ equals **[2009]**

(a) 1 (b) log 2 (c) $-\log 2$ (d) -1

12. Let $f: (-1, 1) \to \mathbf{R}$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = [f(2f(x) + 2)]^2$. Then $g'(0) =$ **[2010]**

(a) -4 (b) 0 (c) -2 (d) 4

13. $\frac{d^2x}{dy^2}$ equals : **[2011]**

(a) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$ (b) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$

(c) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$ (d) $\left(\frac{d^2y}{dx^2}\right)^{-1}$

14. If $y = \sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to : **[JEE M 2013]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) 1 (d) $\sqrt{2}$

15. If g is the inverse of a function f and $f'(x) = \frac{1}{1+x^5}$, then $g'(x)$ is equal to: **[JEE M 2014]**

(a) $\frac{1}{1+\{g(x)\}^5}$ (b) $1+\{g(x)\}^5$

(c) $1+x^5$ (d) $5x^4$

16. If $x = -1$ and $x = 2$ are extreme points of $f(x) = \alpha \log|x| + \beta x^2 + x$ then **[JEE M 2014]**

(a) $\alpha = 2, \beta = -\frac{1}{2}$ (b) $\alpha = 2, \beta = \frac{1}{2}$

(c) $\alpha = -6, \beta = \frac{1}{2}$ (d) $\alpha = -6, \beta = -\frac{1}{2}$

12 Differentiation

Section-A : JEE Advanced/ IIT-JEE

A 1. $\frac{2+2x-2x^2}{(x^2+1)^2}\sin\left(\frac{2x-1}{x^2+1}\right)^2$ 2. zero 3. $1/e$ 4. 4 5. 1 6. 1

B 1. T

C 1. (c) 2. (b) 3. (a) 4. (b) 5. (c) 6. (a)
7. (d) 8. (d) 9. (a) 10. (b)

D 1. (b, c)

E 1. $2x\cos(x^2+1)$ 2. $\frac{-2}{9}$

3. $\frac{5}{3}\cdot\frac{1}{(1-x)^2}-2\sin(4x+2)$, if $x<1$; $-\frac{5}{3}\cdot\frac{1}{(1-x)^2}-2\sin(4x+2)$, if $x>1$

4. $e^{x\sin x^3}\left[\sin x^3+3x^3\cos x^3\right]+(\tan x)^x\left[\frac{2x}{\sin 2x}+\log\tan x\right]$ 5. 11 8. 0

H 1. (b) 2. (a)

I 1. 2 2. 1

Section-B : JEE Main/ AIEEE

1. (a) 2. (c) 3. (d) 4. (b) 5. (c) 6. (a) 7. (d)
8. (d) 9. (c) 10. (a) 11. (d) 12. (a) 13. (c) 14. (a)
15. (b) 16. (a)

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. $\frac{2+2x-2x^2}{(x^2+1)^2}\sin\left(\frac{2x-1}{x^2+1}\right)^2$

Given: $y=f\left(\frac{2x-1}{x^2+1}\right); f'(x)=\sin x^2$

$$\therefore \frac{dy}{dx}=f'\left(\frac{2x-1}{x^2+1}\right)\cdot\frac{d}{dx}\left(\frac{2x-1}{x^2+1}\right)$$

$$=\sin\left(\frac{2x-1}{x^2+1}\right)^2\cdot\frac{2(x^2+1)-2x(2x-1)}{(x^2+1)^2}$$

$$=\frac{2+2x-2x^2}{(x^2+1)^2}\sin\left(\frac{2x-1}{x^2+1}\right)^2$$

2. Given that $F(x)=\begin{vmatrix} f_1(x) & f_2(x) & f_3(x)\\ g_1(x) & g_2(x) & g_3(x)\\ h_1(x) & h_2(x) & h_3(x)\end{vmatrix}$ ...(1)

Where $f_r(x), g_r(x), h_r(x), r=1,2,3$, are polynominals in x and hence differentiable and

$f_r(a)=g_r(a)=h_r(a), r=1,2,3$ ...(2)

Differentiating eq. (1) with respect to x, we get

$$F'(x)=\begin{vmatrix} f_1'(x) & f_2'(x) & f_3'(x)\\ g_1(x) & g_2(x) & g_3(x)\\ h_1(x) & h_2(x) & h_3(x)\end{vmatrix}$$

$$+\begin{vmatrix} f_1(x) & f_2(x) & f_3(x)\\ g_1'(x) & g_2'(x) & g_3'(x)\\ h_1(x) & h_2(x) & h_3(x)\end{vmatrix}+\begin{vmatrix} f_1(x) & f_2(x) & f_3(x)\\ g_1(x) & g_2(x) & g_3(x)\\ h_1'(x) & h_2'(x) & h_3'(x)\end{vmatrix}$$

$$\therefore F'(a)=\begin{vmatrix} f_1'(a) & f_2'(a) & f_3'(a)\\ g_1(a) & g_2(a) & g_3(a)\\ h_1(a) & h_2(a) & h_3(a)\end{vmatrix}$$

$$+\begin{vmatrix} f_1(a) & f_2(a) & f_3(a) \\ g_1'(a) & g_2'(a) & g_3'(a) \\ h_1(a) & h_2(a) & h_3(a) \end{vmatrix} + \begin{vmatrix} f_1(a) & f_2(a) & f_3(a) \\ g_1(a) & g_2(a) & g_3(a) \\ h_1'(a) & h_2'(a) & h_3'(a) \end{vmatrix}$$

$F'(a) = D_1 + D_2 + D_3$

Using eq. (2) we get $D_1 = D_2 = D_3 = 0$ [By the property of determinants that $D = 0$ if two rows in D are identical]

$\therefore \quad F'(a) = 0.$

3. Given that

$$f(x) = \log_x(\ln x) = \frac{\log_e(\log_e x)}{(\log_e x)}$$

$$f'(x) = \frac{\frac{1}{\log_e x} \times \frac{1}{x} \times \log_e x - \frac{1}{x}\log_e(\log_e x)}{(\log_e x)^2}$$

$$= \frac{\frac{1}{x}[1 - \log_e(\log_e x)]}{(\log_x x)^2}$$

$$f'(e) = \frac{\frac{1}{e}[1 - \log_e(\log_e e)]}{(\log_e e)^2} = \frac{\frac{1}{e}[1 - \log_e 1]}{(1)^2} = \frac{1}{e}(1 - 0) = \frac{1}{e}.$$

4. Let $u = \sec^{-1}\left(\frac{1}{2x^2 - 1}\right); v = \sqrt{1 - x^2}$

Then to find $\left.\frac{du}{dv}\right|_{x=1/2}$, we have

$u = \cos^{-1}(2x^2 - 1) = 2\cos^{-1} x$

$\therefore \quad \frac{du}{dx} = \frac{-2}{\sqrt{1 - x^2}}$ and $\quad v = \sqrt{1 - x^2}$

$\therefore \quad \frac{dv}{dx} = \frac{-x}{\sqrt{1 - x^2}} \quad \therefore \quad \frac{du}{dv} = \frac{\frac{-2}{\sqrt{1-x^2}}}{\frac{-x}{\sqrt{1-x^2}}} = \frac{2}{x}$

$\therefore \quad \left.\frac{du}{dv}\right|_{x=\frac{1}{2}} = 4$

5. $f(x) = |x - 2|$

$\Rightarrow \quad g(x) = f(f(x)) = |f(x) - 2|$ as $x > 20$

$= ||x - 2| - 2| = |x - 2 - 2|$ as $x > 20$

$= |x - 4| = x - 4$ as $x > 20 \qquad \therefore \quad g'(x) = 1$

6. Given : $xe^{xy} = y + \sin^2 x$

Differentiating both sides w. r.to x, we get

$$e^{xy}.1 + xe^{xy}\left(y + x\frac{dy}{dx}\right) = \frac{dy}{dx} + 2\sin x \cos x$$

Put $x = 0 \Rightarrow \; 1 + 0 = \frac{dy}{dx} + 0 \;\Rightarrow\; \frac{dy}{dx} = 1$

B. True/ False

1. Consider $\phi(x) = \frac{f(x) + f(-x)}{2}$, which is an even function

Now, $\psi(x) = \phi'(x) = \frac{f'(x) - f'(-x)}{2}$

$\psi(-x) = \frac{f'(-x) - f'(x)}{2} = -\psi(x) \;\; \therefore \;\; \psi$ is odd.

C. MCQs with ONE Correct Answer

1. (c) We have $y^2 = P(x)$, ...(1)

where $P(x)$ is a polynomial of degree 3 and hence thrice differentiable. Differentiating (1) w.r. to x, we get

$$2y\frac{dy}{dx} = P'(x) \qquad ...(2)$$

Again differentiating with respect to x, we get

$$2\left(\frac{dy}{dx}\right)^2 + 2y\frac{d^2y}{dx^2} = P''(x)$$

$$\Rightarrow \frac{[P'(x)]^2}{2y^2} + 2y\frac{d^2y}{dx^2} = P''(x) \qquad \text{[Using (2)]}$$

$$\Rightarrow 4y^3\frac{d^2y}{dx^2} = 2y^2P''(x) - [P'(x)]^2$$

$$\Rightarrow 4y^3\frac{d^2y}{dx^2} = 2P(x)P''(x) - [P'(x)]^2 \quad \text{[Using (1)]}$$

$$\Rightarrow 2y^3\frac{d^2y}{dx^2} = P(x)P''(x) - \frac{1}{2}[P'(x)]^2$$

Again differentiating w.r. to x, we get $2\frac{d}{dx}\left(y^3\frac{d^2y}{dx^2}\right)$

$= P'''(x)P(x) + P''(x)P'(x) - P'(x)P''(x) = P'''(x)P(x)$

2. (b) Let $f(x) = ax^2 + bx + c$

As given that $f(x) > 0, \; \forall \; x \in R$

$\therefore \quad a > 0$ and $D < 0$

$\Rightarrow \quad a > 0$ and $b^2 - 4ac < 0$ .....(1)

Now, $g(x) = f(x) + f'(x) + f''(x)$

$= ax^2 + bx + c + 2ax + b + 2a$

$= ax^2 + (2a + b)x + (2a + b + c)$

Here, $D = (2a + b)^2 - 4a(2a + b + c)$

$= 4a^2 + b^2 + 4ab - 8a^2 - 4ab - 4ac$

$= b^2 - 4a^2 - 4ac = -4a^2 + b^2 - 4ac$

$= (-ve) + (-ve) = -ve$ [Using eq. (1)]

Also $a > 0$ from (1),

$\therefore \quad g(x) > 0, \; \forall \; x \in R$

3. (a) $y = (\sin x)^{\tan x} \Rightarrow \log y = \tan x.\log\sin x$

Differentiating w.r.t. x,

$$\frac{1}{y}\frac{dy}{dx} = \sec^2 x \log\sin x + \tan x.\frac{1}{\sin x}.\cos x$$

$$\Rightarrow \frac{dy}{dx} = (\sin x)^{\tan x}[1 + \sec^2 x \log\sin x]$$

4. (b) $x^2 + y^2 = 1 \Rightarrow 2x + 2yy' = 0 \Rightarrow x + yy' = 0$

$\Rightarrow \quad 1 + yy'' + (y')^2 = 0 \Rightarrow yy'' + (y')^2 + 1 = 0$

5. **(c)** $F(x) = \int_0^x f(t)dt$ and $F(x^2) = x^2(1+x)$

$F'(x) = f(x)$ ...... (1)

But $F'(x^2) . 2x = 2x + 3x^2$

$\Rightarrow F'(x^2) = \left(\frac{2+3x}{2}\right) \Rightarrow f(x^2) = \frac{2+3x}{2}$

$\Rightarrow f(4) = \frac{2+3\times 2}{2} = \frac{8}{2} = 4$

6. **(a)** $\log(x+y) = 2xy$ when $x = 0$ then $y = 1$

Differentiating w.r.t. x

$$\frac{1}{x+y}\left[1+\frac{dy}{dx}\right] = 2y + \frac{2xdy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{1}{x+y} - 2y}{2x - \frac{1}{x+y}} \Rightarrow y'(0) = \frac{1-2}{0-1} = 1$$

7. **(d)** Let us consider the function $g(x) = f(x) - x^2$ so that

$g(1) = f(1) - 1^2 = 1 - 1 = 0$
$g(2) = f(2) - 2^2 = 4 - 4 = 0$
$g(3) = f(3) - 3^2 = 9 - 9 = 0$

Since $f(x)$ is twice differentiable we can say $g(x)$ is continuous and differentiable everywhere and

$g(1) = g(2) = g(3) = 0$

$\therefore$ By Rolle's theorem, $g'(c) = 0$ for some $c \in (1,2)$

and $g'(d) = 0$ for some $d \in (2,3)$

Again by Rolle's theorem,

$g''(e) = 0$ for some $e \in (c,d) \Rightarrow e \in (1,3)$

$\Rightarrow f''(e) - 2 = 0$ or $f''(e) = 2$ for some $x \in (1,3)$

$f''(x) = 2$ for some $x \in (1,3)$

8. **(d)** $\frac{d^2x}{d^2y} = \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dx}\left(\frac{dx}{dy}\right) \times \frac{dx}{dy}$

$$= \left\{\frac{d}{dx}\left[\frac{1}{\left(\frac{dy}{dx}\right)}\right]\right\} \times \frac{1}{\frac{dy}{dx}} = -\frac{1}{\left(\frac{dy}{dx}\right)^2} \times \frac{d^2y}{dx^2} \times \frac{1}{\left(\frac{dy}{dx}\right)}$$

$$= -\left(\frac{dy}{dx}\right)^{-3} \frac{d^2y}{dx^2}$$

9. **(a)** Given that $g(x) = \log f(x) \Rightarrow g(x+1) = \log f(x+1)$

$\Rightarrow g(x+1) = \log x f(x)$ $[\because f(x+1) = x f(x)]$

$\Rightarrow g(x+1) = \log x + \log f(x) \Rightarrow g(x+1) - g(x) = \log(x)$

$\Rightarrow g'(x+1) - g'(x) = \frac{1}{x}$

$\Rightarrow g''(x+1) - g''(x) = -\frac{1}{x^2}$

Putting, $x = x - \frac{1}{2}$, we get

$$\Rightarrow g''\left(x+\frac{1}{2}\right) - g''\left(x-\frac{1}{2}\right) = -\frac{1}{\left(x-\frac{1}{2}\right)^2} = \frac{-2^2}{(2x-1)^2}$$

Putting $x = 1, 2, 3$ ...., N we get

$g''\left(\frac{3}{2}\right) - g''\left(\frac{1}{2}\right) = -\frac{2^2}{1^2}$ ...(1)

$g''\left(\frac{5}{2}\right) - g''\left(\frac{3}{2}\right) = \frac{-2^2}{3^2}$ ...(2)

$g''\left(\frac{7}{2}\right) - g''\left(\frac{5}{2}\right) = \frac{-2^2}{5^2}$ ...(3)

$g''\left(N+\frac{1}{2}\right) - g''\left(N-\frac{1}{2}\right) = -\frac{2^2}{(2N-1)^2}$ ...(N)

Adding all the above equations, we get

$$g''\left(N+\frac{1}{2}\right) - g''\left(\frac{1}{2}\right) = -4\left[1+\frac{1}{3^2}+\frac{1}{5^2}+.....+\frac{1}{(2N-1)^2}\right]$$

10. **(b)** $F(x) = \int_0^{x^2} f\left(\sqrt{t}\right)dt$ for $x \in [0,2]$

$\Rightarrow F'(x) = f(x).2x$

Now $F'(x) = f'(x) \forall x \in (0,2)$

$\Rightarrow f(x).2x = f'(x) \Rightarrow \frac{f'(x)}{f(x)} = 2x$

$\Rightarrow \ln f(x) = x^2 + c \Rightarrow f(x) = e^{x^2+c} = e^{x^2}. e^c$

As $f(0) = 1 \Rightarrow 1 = e^c$

$\therefore f(x) = e^{x^2}$

So $F(x) = \int_0^{x^2} e^x dx = e^{x^2} - 1$ $\therefore F(2) = e^4 - 1$

D. MCQs with ONE or MORE THAN one Correct

1. **(b, c)** $f(x) = x^3 + 3x + 2 \Rightarrow f'(x) = 3x^2 + 3$

Also $f(0) = 2, f(1) = 6, f(2) = 16, f(3) = 38, f(6) = 236$

And $g(f(x)) = x \Rightarrow g(2) = 0, g(6) = 1, g(16) = 2, g(38) = 3, g(236) = 6$

(a) $g(f(x)) = x \Rightarrow g'(f(x)). f'(x) = 1$

For $g'(2)$, $f(x) = 2 \Rightarrow x = 0$

$\therefore$ Putting $x = 0$, we get $g'(f(0))\ f'(0) = 1$

$\Rightarrow g'(2) = \frac{1}{3}$

(b) $h(g(g(x))) = x \Rightarrow h'(g(g(x))). g'(g(x)). g'(x) = 1$

For $h'(1)$, we need $g(g(x)) = 1$

$\Rightarrow g(x) = 6 \Rightarrow x = 236$

$\therefore$ Putting $x = 236$, we get

$$h'[g(g(236))]=\frac{1}{g'(g(236)).g'(236)}$$

$$\Rightarrow h'(g(6))=\frac{1}{g'(6)\,g'(236)}$$

$$\Rightarrow h'(1)=\frac{1}{g'(f(1))\,g'(f(6))}=f'(1)\,f'(6)$$

$=6\times 111=666$

(c) $h[g(g(x))]=x$

For h (0), $g(g(x))=0\Rightarrow g(x)=2\Rightarrow x=16$

$\therefore$ Putting $x=16$, we get

$h(g(g(16)))=16$

$\Rightarrow h(0)=16$

(d) $h[g(g(x))]=x$

For $h(g(3))$, we need $g(x)=3\Rightarrow x=38$

$\therefore$ Putting $x=38$, we get

$h[g(g(38))]=38\Rightarrow h(g(3))=38$

E. Subjective Problems

1. Let $f(x)=\sin(x^2+1)$ then

$$f'(x)=\lim_{\delta x\to 0}\frac{\sin[(x+\delta x)^2+1]-\sin[x^2+1]}{\delta x}$$

$$\Rightarrow f'(x)=\lim_{\delta x\to 0}2\cos\left(\frac{x^2+(\delta x)^2+2x\delta x+1+x^2+1}{2}\right).\frac{\sin\left(\frac{x^2+(\delta x)^2+2x\delta x+1-x^2-1}{2}\right)}{\delta x}$$

$$=\lim_{\delta x\to 0}\frac{2\cos\left[x^2+1+x\delta x+\frac{(\delta x)^2}{2}\right]\sin\left[\frac{(\delta x)^2+2x\delta x}{2}\right]}{\delta x\left[\frac{\delta x+2x}{2}\right]}\times\left(\frac{\delta x+2x}{2}\right)$$

$$=2\cos(x^2+1)\lim_{\delta x\to 0}\frac{\sin\left[\frac{(\delta x)^2+2x\delta x}{2}\right]}{\left[\frac{(\delta x)^2+2x\delta x}{2}\right]}\times\left(\frac{\delta x+2x}{2}\right)$$

$$=2\cos(x^2+1)\times 1\times\frac{2x}{2}=2x\cos(x^2+1)$$

2. $$f(x)\begin{cases}\dfrac{x-1}{2x^2-7x+5}, & x\neq 1\\ -\dfrac{1}{3}, & x=1\end{cases}$$

$$\therefore\quad f'(x)|_{x=1}=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

$$=\lim_{h\to 0}\left[\frac{\frac{1+h-1}{2(1+h)^2-7(1+h)+5}+\frac{1}{3}}{h}\right]=\lim_{h\to 0}\frac{\frac{h}{2h^2-3h}+\frac{1}{3}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{1}{2h-3}+\frac{1}{3}}{h}=\lim_{h\to 0}\frac{2h}{3h(2h-3)}=\lim_{h\to 0}\frac{2}{3(2h-3)}$$

$=-2/9$

3. We have, $y=\dfrac{5x}{3|1-x|}+\cos^2(2x+1)$

(Clearly y is not defined at $x=1$)

$$\Rightarrow y=\begin{cases}\dfrac{5x}{3(1-x)}+\cos^2(2x+1), & x<1\\ \dfrac{5x}{3(x-1)}+\cos^2(2x+1), & x>1\end{cases}$$

$$\Rightarrow\frac{dy}{dx}=\begin{cases}\dfrac{5}{3}\left(\dfrac{(1-x)-x(-1)}{(1-x)^2}\right)-2\sin(4x+2), & x<1\\ \dfrac{5}{3}\left(\dfrac{(x-1)-x}{(x-1)^2}\right)-2\sin(4x+2), & x>1\end{cases}$$

or $$\frac{dy}{dx}=\begin{cases}\dfrac{5}{3}\dfrac{1}{(1-x)^2}-2\sin(4x+2), & x<1\\ -\dfrac{5}{3}\dfrac{1}{(x-1)^2}-2\sin(4x+2), & x>1\end{cases}$$

4. We are given $y=e^{x\sin x^3}+(\tan x)^x$

Here y is the sum of two functions and in the second function base as well as power are functions of x. Therefore we will use logarithmic differentiation here.

Let $y=u+v$

where $u=e^{x\sin x^3}$ ...(1)

and $v=(\tan x)^x$ ...(2)

$$\therefore\quad\frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}\quad ...(3)$$

Differentiating (1) with respect to x, we get

$$\frac{du}{dx}=e^{x\sin x^3}.\frac{d}{dx}(x\sin x^3)$$

$$=e^{x\sin x^3}.[3x^3.\cos x^3+\sin x^3]$$

Taking log on both sides on equ^n (2), we get

$\log v=x\log\tan x$

Differentiating the above with respect to x, we get

$$\frac{1}{v}\frac{dv}{dx}=x.\frac{1}{\tan x}.\sec^2 x+1.\log\tan x$$

$$\therefore\quad\frac{dv}{dx}=(\tan x)^x\left[\frac{2x}{\sin 2x}+\log\tan x\right]$$

Substituting the value of $\frac{du}{dx}$ *and* $\frac{dv}{dx}$ in eqⁿ (3), we get

$$\frac{dy}{dx} = e^{x\sin x^3}[\sin x^3 + 3x^3\cos x^3]$$

$$+(\tan x)^x\left[\frac{2x}{\sin 2x} + \log\tan x\right]$$

5. Given that f is twice differentiable such that

$f''(x) = -f(x)$ and $f'(x) = g(x)$

$h(x) = [f(x)]^2 + [g(x)]^2$

To find $h(10)$ when $h(5) = 11$.

Consider $h'(x) = 2ff' + 2gg' = 2f(x)\,g(x) + 2g(x)f''(x)$

$[\because\ g(x) = f'(x) \Rightarrow g'(x) = f''(x)]$

$= 2f(x)\,g(x) + 2g(x)(-f(x))$

$= 2f(x)\,g(x) - 2f(x)\,g(x) = 0$

$\therefore\ h'(x) = 0, \forall x$

$\Rightarrow$ h is a constant function

$\therefore\ h(5) = 11 \Rightarrow h(10) = 11.$

6. Let $F(x) = \begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$

Given that α is a repeated root of quadratic equation $f(x) = 0$

$\therefore$ We must have $f(x) = k(x-\alpha)^2$; where k is a non- zero real no.

If we put $x = \alpha$ on both sides of eq. (1); we get

$$F(\alpha) = \begin{vmatrix} A(\alpha) & B(\alpha) & C(\alpha) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix} = 0$$

$[\because\ R_1$ and R_2 are identical]

$\therefore\ F(\alpha) = 0$

Hence $(x-\alpha)$ is a factor of $F(x)$

Differentiating eq. (1) w.r. to x, we get

$$F'(x) = \begin{vmatrix} A'(x) & B'(x) & C'(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$$

Putting $x = \alpha$ on both sides, we get

$$F'(\alpha) = \begin{vmatrix} A'(\alpha) & B'(\alpha) & C'(\alpha) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix} = 0$$

[as R_1 and R_3 are identical]

$\Rightarrow$ $(x-\alpha)$ is a factor of $F'(x)$ also. Or we can say $(x-\alpha)^2$ is a factor of $F(x)$.

$\Rightarrow$ $F(x)$ is divisible by $f(x)$.

7. We have, $x = \sec\theta - \cos\theta$, $y = \sec^n\theta - \cos^n\theta$

$$\Rightarrow \frac{dx}{d\theta} = \sec\theta\tan\theta + \sin\theta$$

$= \sec\theta\tan\theta + \tan\theta\cos\theta = \tan\theta(\sec\theta + \cos\theta)$

$$\text{and } \frac{dy}{d\theta} = n\sec^{n-1}\theta\sec\theta\tan\theta - n\cos^{n-1}\theta(-\sin\theta)$$

$= n\sec^n\theta\tan\theta + n\tan\theta\cos^n\theta = n\tan\theta(\sec^n\theta + \cos^n\theta)$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{n\tan\theta(\sec^n\theta + \cos^n\theta)}{\tan\theta(\sec\theta + \cos\theta)}$$

$$\text{or } \frac{dy}{dx} = \frac{n(\sec^n\theta + \cos^n\theta)}{(\sec\theta + \cos\theta)} \quad ...(1)$$

Also $x^2 + 4 = (\sec\theta - \cos\theta)^2 + 4$

$= \sec^2\theta + \cos^2\theta - 2\sec\theta\cos\theta + 4$

$= \sec^2\theta + \cos^2\theta + 2$

$= (\sec\theta + \cos\theta)^2$ ...(2)

and $y^2 + 4 = \sec^n\theta - \cos^n\theta)^2 + 4$

$= \sec^{2n}\theta + \cos^{2n}\theta - 2\sec^n\theta\cos^n\theta + 4$

$= \sec^{2n}\theta + \cos^{2n}\theta + 2$

$= (\sec^n\theta + \cos^n\theta)^2$ ...(3)

Now we have to prove

$$(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$$

$$\text{LHS} = (\sec\theta + \cos\theta)^2 \cdot \frac{n^2(\sec^n\theta + \cos^n\theta)^2}{(\sec\theta + \cos\theta)^2}$$

[Using (1) and (2)]

$= n^2(\sec^n\theta + \cos^n\theta)^2$

$= n^2(y^2+4)$ [From eq. (3)]

$=$ RHS

8. We have given the function

$$(\sin y)^{\sin\left(\frac{\pi x}{2}\right)} + \frac{\sqrt{3}}{2}\sec^{-1}(2x) + 2^x\tan[\ln(x+2)] = 0 \quad ...(1)$$

For $x = -1$, we have

$$(\sin y)^{\sin\left(-\frac{\pi}{2}\right)} + \frac{\sqrt{3}}{2}\sec^{-1}(-2) + 2^{-1}\tan[\ln(-1+2)] = 0$$

$$\Rightarrow (\sin y)^{-1} + \frac{\sqrt{3}}{2}\left(\frac{2\pi}{3}\right) + \frac{1}{2}\tan 0 = 0 \Rightarrow \frac{1}{\sin y} = -\frac{\pi}{\sqrt{3}}$$

$$\Rightarrow \sin y = -\frac{\sqrt{3}}{\pi}, \text{ when } x = -1 \quad ....(2)$$

Now Let $u = (\sin y)^{\sin\left(\frac{\pi x}{2}\right)}$

Taking ln on both sides; we get

$$\ln u = \sin\left(\frac{\pi x}{2}\right)\ln\sin y$$

Differentiating both sides with respect to x, we get

$$\frac{1}{u}\frac{du}{dx} = \frac{\pi}{2}\cos\left(\frac{\pi x}{2}\right)\ln\sin y + \cot y\frac{dy}{dx}\sin\left(\frac{\pi x}{2}\right)$$

$$\Rightarrow \frac{du}{dx} = (\sin y)^{\sin\left(\frac{\pi x}{2}\right)}$$

$$\times\left[\frac{\pi}{2}\cos\left(\frac{\pi x}{2}\right)\ln\sin y + \sin\left(\frac{\pi x}{2}\right)\cot y\frac{dy}{dx}\right] \quad ...(3)$$

Now differentiating eq. (1), we get

$$\frac{d}{dx}\left[(\sin y)^{\sin\left(\frac{\pi x}{2}\right)}\right] + \frac{\sqrt{3}}{2}\frac{1}{2x\sqrt{4x^2-1}}.2$$

$$+ 2^x(\ln 2)\tan[(\ln(x+2)]$$

$$+2^x\sec^2[\ln(x+2)]\frac{1}{x+2} = 0$$

$$\Rightarrow (\sin y)^{\sin\left(\frac{\pi x}{2}\right)}\left[\frac{\pi}{2}\cos\left(\frac{\pi x}{2}\right)\ln\sin y\right.$$

$$\left. + \sin\left(\frac{\pi x}{2}\right)\cot y\frac{dy}{dx}\right]$$

$$+ \frac{\sqrt{3}}{2x\sqrt{4x^2-1}} + 2^x\ln 2\tan(\ln(x+2))$$

$$+ \frac{2^x\sec^2[\ln(x+2)]}{x+2} = 0$$

At $x = -1$ and $\sin y = -\frac{\sqrt{3}}{\pi}$, we get

$$\Rightarrow \left(-\frac{\sqrt{3}}{\pi}\right)^{-1}\left[0-(-1)\sqrt{\frac{\pi^2}{3}-1}\left(\frac{dy}{dx}\right)_{x=-1}\right]$$

$$+\frac{\sqrt{3}}{-2\sqrt{3}} + 0 + 2^{-1} = 0$$

$$\Rightarrow -\frac{\pi}{\sqrt{3}\sqrt{3}}\sqrt{\pi^2-3}\left(\frac{dy}{dx}\right)_{x=-1} - \frac{1}{2} + \frac{1}{2} = 0 \Rightarrow \left.\frac{dy}{dx}\right]_{x=-1} = 0$$

9. $$y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{x-c} + 1$$

$$= \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{x}{x-c}$$

$$= \frac{ax^2}{(x-a)(x-b)(x-c)} + \left(\frac{b}{x-b}+1\right)\frac{x}{x-c}$$

$$= \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{x^2}{(x-b)(x-c)}$$

$$= \left(\frac{a}{x-a}+1\right)\frac{x^2}{(x-b)(x-c)} = \frac{x^3}{(x-a)(x-b)(x-c)}$$

$\Rightarrow \log y = 3\log x - \log(x-a) - \log(x-b) - \log(x-c)$

$$\Rightarrow \frac{y'}{y} = \frac{3}{x} - \frac{1}{x-a} - \frac{1}{x-b} - \frac{1}{x-c}$$

$$= \left(\frac{1}{x} - \frac{1}{x-a}\right) + \left(\frac{1}{x} - \frac{1}{x-b}\right) + \left(\frac{1}{x} - \frac{1}{x-c}\right)$$

$$= \frac{a}{x(a-x)} + \frac{b}{x(b-x)} + \frac{c}{x(c-x)}$$

$$= \frac{1}{x}\left[\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right]$$

H. Assertion & Reason Type Questions

1. **(b)** Given that $f(x) = 2 + \cos x$ which is continuous and differentiable every where.

Also $f'(x) = -\sin x \Rightarrow f'(x) = 0 \Rightarrow x = n\pi$

$\Rightarrow$ There exists $c \in [t, t+\pi]$ for $t \in R$

Such that $f'(c) = 0$

$\therefore$ Statement-1 is true.

Also $f(x)$ being periodic of period 2π, statement-2 is true, but statement-2 is not a correct explanation of statement-1.

2. **(a)** We have $f(x) = g(x)\sin x$

$\Rightarrow f'(x) = g'(x)\sin x + g(x)\cos x$

$\Rightarrow f'(0) = g'(0)\times 0 + g(0) = g(0) \quad [\because g'(0) = 0]$

$\therefore$ Statement 2 is correct.

$$\text{Also } f''(0) = \lim_{x\to 0}\frac{f'(x) - f'(0)}{x}$$

$$= \lim_{x\to 0}\frac{g(x)\cos x + g'(x)\sin x - g(0)}{x}$$

$$= \lim_{x\to 0}\frac{g(x)\cos x - g(0)}{x} + \lim_{x\to 0}\frac{g'(x)\sin x}{x}$$

$$= \lim_{x\to 0}\frac{g(x)\cos x - g(0)}{x\times\frac{\sin x}{x}} + \lim_{x\to 0}g'(x)$$

$$= \lim_{x\to 0}\frac{g(x)\cos x - g(0)}{\sin x} + g'(0)$$

$$= \lim_{x\to 0}[g(x)\cot(x) - g(0)\text{cosec}\,x] + 0$$

$$= \lim_{x\to 0}[g(x)\cot x - g(0)\text{cosec}\,x]$$

$\therefore$ Statement 1 is also true and is a correct explanation for statement 2.

I. Integer Value Correct Type

1. **(2)** Given that $f(x) = x^3 + e^{x/2}$ and $g(x) = f^{-1}(x)$

then we should have $gof(x) = x$

$\Rightarrow g(f(x)) = x \Rightarrow g(x^3 + e^{x/2}) = x$

Differentiating both sides with respect to x, we get

$$g'(x^3 + e^{x/2}).\left(3x^2 + e^{x/2}.\frac{1}{2}\right) = 1$$

$$\Rightarrow g'(x^3 + e^{x/2}) = \frac{1}{3x^2 + e^{x/2}.\frac{1}{2}}$$

For $x = 0$, we get $g'(1) = \frac{1}{1/2} = 2$

2. **(1)** $f(\theta)=\sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$

$=\sin\left[\sin^{-1}\left(\frac{\sin\theta}{\sqrt{\sin^2\theta+\cos 2\theta}}\right)\right]\left[\because \tan^{-1}\frac{x}{y}=\sin^{-1}\frac{x}{\sqrt{x^2+y^2}}\right]$

$=\sin\left[\sin^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)\right]=\tan\theta$

$\therefore \frac{df(\theta)}{d\tan\theta}=1.$

Section-B JEE Main/ AIEEE

1. **(a)** $y=(x+\sqrt{1+x^2})^n$

$\frac{dy}{dx}=n(x+\sqrt{1+x^2})^{n-1}\left(1+\frac{1}{2}(1+x^2)^{-1/2}.2x\right);$

$\frac{dy}{dx}=n(x+\sqrt{1+x^2})^{n-1}\frac{(\sqrt{1+x^2}+x)}{\sqrt{1+x^2}}$

$=\frac{n(\sqrt{1+x^2}+x)^n}{\sqrt{1+x^2}}$

or $\sqrt{1+x^2}\frac{dy}{dx}=ny$ or $\sqrt{1+x^2}y_1=ny$

$(y_1=\frac{dy}{dx})$ Squaring, $(1+x^2)y_1^2=n^2y^2$

Differentiating, $(1+x^2)2y_1y_2+y_1^2.2x=n^2.2yy_1$

or $(1+x^2)y_2+xy_1=n^2y$

2. **(c)** $F(t)=\int_0^t f(t-y)g(y)dy$

$=\int_0^t e^{t-y}ydy=e^t\int_0^t e^{-y}ydy$

$=e^t\left[-ye^{-y}-e^{-y}\right]_0^t=-e^t\left[ye^{-y}+e^{-y}\right]_0^t$

$=-e^t\left[te^{-t}+e^{-t}-0-1\right]=-e^t\left[\frac{t+1-e^t}{e^t}\right]$

$=e^t-(1+t)$

3. **(d)** $f(x)=x^n \Rightarrow f(1)=1$

$f'(x)=nx^{n-1}\Rightarrow f'(1)=n$

$f''(x)=n(n-1)x^{n-2}\Rightarrow f''(1)=n(n-1)$

....

.... $f^n(x)=n!\Rightarrow f^n(1)=n!$

$=1-\frac{n}{1!}+\frac{n(n-1)}{2!}-\frac{n(n-1)(n-2)}{3!}+....+(-1)^n\frac{n!}{n!}$

$={}^nC_0-{}^nC_1+{}^nC_2-{}^nC_3+......+(-1)^n\,{}^nC_n=0$

4. **(b)** $f(x)=ax^2+bx+c$

$f(1)=f(-1)\Rightarrow a+b+c=a-b+c$ or $b=0$

$\therefore f(x)=ax^2+c$ or $f'(x)=2ax$

Now $f'(a); f'(b)$; and $f'(c)$ are $2a(a); 2a(b); 2a(c)$ i.e. $2a^2, 2ab, 2ac$.

$\Rightarrow$ If a,b,c are in A.P. then $f'(a); f'(b)$ and $f'(c)$ are also in A.P.

5. **(c)** $x=e^{y+e^{y+.....\infty}}\Rightarrow x=e^{y+x}$.

Taking log.

$\log x=y+x\Rightarrow\frac{1}{x}=\frac{dy}{dx}+1\Rightarrow\frac{dy}{dx}=\frac{1}{x}-1=\frac{1-x}{x}$

6. **(a)** $x^2-(a-2)x-a-1=0$

$\Rightarrow \alpha+\beta=a-2;\ \alpha\beta=-(a+1)$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=a^2-2a+6=(a-1)^2+5$

For min. value of $\alpha^2+\beta^2$ where α is an integer

$\Rightarrow a=1.$

7. **(d)** Let α, $\alpha+1$ be roots

Then $\alpha+\alpha+1=b=$ sum of roots $\alpha(\alpha+1)=c$ = product of roots

$\therefore\ b^2-4c=(2\alpha+1)^2-4\alpha(\alpha+1)=1.$

8. (d) $\lim_{x\to 2}\int_0^{f(x)}\frac{4t^3}{x-2}dt = \lim_{x\to 0}\frac{\int_0^{f(x)}4t^3\,dt}{x-2}$

Applying L Hospital rule

$$\lim_{x\to 2}\frac{[4f(x)^3 f'(x)]}{1} = 4(f(2))^3 f'(2) = 4\times 6^3\times\frac{1}{48} = 18$$

9. (c) $f(x)=\begin{cases}\dfrac{x}{1-x}, & x<0\\ \dfrac{x}{1+x}, & x\ge 0\end{cases}$

$$\Rightarrow\ f'(x)=\begin{cases}\dfrac{x}{(1-x)^2}, & x<0\\ \dfrac{x}{(1+x)^2}, & x\ge 0\end{cases}$$

$\therefore$ $f'(x)$ exist at everywhere.

10. (a) $x^m.y^n = (x+y)^{m+n}$

$\Rightarrow$ $m\ln x + n\ln y = (m+n)\ln(x+y)$

Differentiating both sides.

$$\therefore\ \frac{m}{x}+\frac{n}{y}\frac{dy}{dx}=\frac{m+n}{x+y}\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow\ \left(\frac{m}{x}-\frac{m+n}{x+y}\right)=\left(\frac{m+n}{x+y}-\frac{n}{y}\right)\frac{dy}{dx}$$

$$\Rightarrow\ \frac{my-nx}{x(x+y)}=\left(\frac{my-nx}{y(x+y)}\right)\frac{dy}{dx}\Rightarrow\ \frac{dy}{dx}=\frac{y}{x}$$

11. (d) $x^{2x}-2x^x\cot y-1=0$

$\Rightarrow 2\cot y = x^x - x^{-x} \Rightarrow 2\cot y = u-\frac{1}{u}$ where $u=x^x$

Differentiating both sides with respect to x, we get

$$\Rightarrow\ -\ 2\text{cosec}^2 y\frac{dy}{dx}=\left(1+\frac{1}{u^2}\right)\frac{du}{dx}$$

where $u=x^x \Rightarrow \log u = x\log x$

$$\Rightarrow\ \frac{1}{u}\frac{du}{dx}=1+\log x\ \Rightarrow\ \frac{du}{dx}=x^x(1+\log x)$$

$$\therefore\ \text{We get} -2\,\text{cosec}^2 y\frac{dy}{dx}=(1+x^{-2x}).x^x(1+\log x)$$

$$\Rightarrow\ \frac{dy}{dx}=\frac{\left(x^x+x^{-x}\right)(1+\log x)}{-2(1+\cot^2 y)}\ \ ....(i)$$

Now when $x=1$, $x^{2x}-2x^x\cot y-1=0$, gives

$1-2\cot y-1=0 \Rightarrow\ \ \cot y=0$

$\therefore$ From equation (i), at $x=1$ and $\cot y=0$, we get

$$y'(1)=\frac{(1+1)(1+0)}{-2(1+0)}=-1$$

12. (a) $g'(x)=2\left(f(2f(x)+2)\right)\left(\frac{d}{dx}\left(f(2f(x)+2)\right)\right)$

$=2f(2f(x)+2)f'(2f(x))+2).(2f'(x))$

$\Rightarrow g'(0)=2f(2f(0)+2).f'(2f(0)+2).2f'(0)=4f(0)(f'(0))^2$

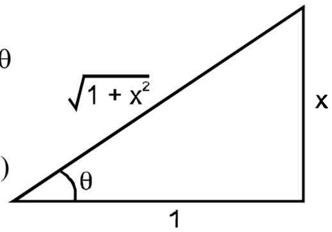
$=4(-1)(1)^2=-4$

13. (c) $\frac{d^2x}{dy^2}=\frac{d}{dy}\left(\frac{dx}{dy}\right)=\frac{d}{dx}\left(\frac{dx}{dy}\right)\frac{dx}{dy}$

$$=\frac{d}{dx}\left(\frac{1}{dy/dx}\right)\frac{dx}{dy}=-\frac{1}{\left(\frac{dy}{dx}\right)^2}\cdot\frac{d^2y}{dx^2}\cdot\frac{1}{\frac{dy}{dx}}=-\frac{1}{\left(\frac{dy}{dx}\right)^3}\frac{d^2y}{dx^2}$$

14. (a) Let $y=\sec(\tan^{-1}x)$ and $\tan^{-1}x=\theta$.

$\Rightarrow\ x=\tan\theta$



Thus, we have $y=\sec\theta$

$\Rightarrow\ y=\sqrt{1+x^2}$

$(\because\ \ \sec^2\theta=1+\tan^2\theta)$

$$\Rightarrow\ \frac{dy}{dx}=\frac{1}{2\sqrt{1+x^2}}\cdot 2x$$

At $x=1$, $\frac{dy}{dx}=\frac{1}{\sqrt{2}}$.

15. (b) Since $f(x)$ and $g(x)$ are inverse of each other

$$\therefore\ g'(f(x))=\frac{1}{f'(x)}$$

$\Rightarrow\ g'(f(x))=1+x^5$ $\left(\because\ f'(x)=\frac{1}{1+x^5}\right)$

Here $x=g(y)$

$\therefore\ \ g'(y)=1+\{g(y)\}^5$

$\Rightarrow\ \ g'(x)=1+\{g(x)\}^5$

16. (a) Let $f(x)=\alpha\log|x|+\beta x^2+x$

Differentiating both sides,

$$f'(x)=\frac{\alpha}{x}+2\beta x+1$$

Since $x=-1$ and $x=2$ are extreme points therefore $f'(x)=0$ at these points.

Put $x=-1$ and $x=2$ in $f'(x)$, we get

$-\alpha-2\beta+1=0\Rightarrow\alpha+2\beta=1$ ...(i)

$\frac{\alpha}{2}+4\beta+1=0\Rightarrow\alpha+8\beta=-2$ ...(ii)

On solving (i) and (ii), we get

$6\beta=-3\Rightarrow\beta=-\frac{1}{2}$ $\quad\therefore\ \ \alpha=2$